

TERRAMETRA

COMPLEX NUMBERS

Terrametra Resources

Lynn Patten



- Basic Concepts
- Operations on Complex Numbers



There is no real number solution of the equation $x^2 = -1$ since no real number, when squared, gives -1.

To extend the real number system to include solutions of equations of this type, the number *i* is defined to have the following property ...

$$i = \sqrt{-1}$$
 therefore $i^2 = -1$



If a and b are real numbers, then any number of the form a + bi is a <u>complex number</u>.

In the complex number a + bi, *a* is the <u>*real part*</u> and *b* is the <u>*imaginary part*</u>.



Two complex numbers a + bi and c + di are equal, provided that their real parts are equal and their imaginary parts are equal, ...

a + bi = c + di if and only if a = c and b = d



For complex number a + bi, if b = 0, then a + bi = a, ...

Thus, the set of real numbers is a subset of the set of complex numbers.



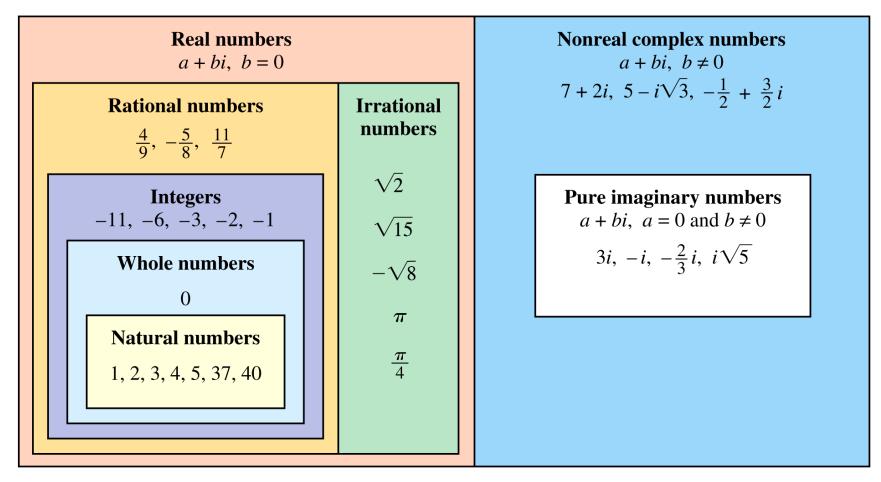
If a = 0 and $b \neq 0$, the complex number is said to be a *pure imaginary number*.

A pure imaginary number, or a number like 7 + 2i with $a \neq 0$ and $b \neq 0$, is a *nonreal complex number*.

A complex number written in the form a + bi (or a + ib) is in <u>standard form</u>.



Complex Numbers *a* + *bi*, for *a* and *b* Real





THE EXPRESSION $\sqrt{-a}$

THE EXPRESSION $\sqrt{-a}$

If a > 0, then $\sqrt{-a} = i\sqrt{a}$.



1(a) Write as the product of a real number and i, using the definition of $\sqrt{-a}$.

 $\sqrt{-16}$

$$\sqrt{-16} = i\sqrt{16} = 4i$$



1(b) Write as the product of a real number and i, using the definition of $\sqrt{-a}$.

 $\sqrt{-70}$

$$\sqrt{-70} = i\sqrt{70} = i\sqrt{70}$$



1(c) Write as the product of a real number and i, using the definition of $\sqrt{-a}$.

 $\sqrt{-48}$

$$\sqrt{-48} = i\sqrt{48} = i\sqrt{16 \cdot 3} = 4i\sqrt{3}$$



OPERATIONS on COMPLEX NUMBERS

Products or quotients with negative radicands are simplified by first rewriting $\sqrt{-a}$ as $i\sqrt{a}$ for a positive number a, ...

Then the properties of real numbers and the fact that $i^2 = -1$ are applied.



OPERATIONS on COMPLEX NUMBERS

Caution

When working with negative radicands, use the definition $\sqrt{-a} = i\sqrt{a}$ before using any of the other rules for radicals.

In particular, the rule

$$\sqrt{b} \cdot \sqrt{d} = \sqrt{bd}$$

is valid only when b and d are not both negative.

$$\sqrt{-4} \cdot \sqrt{-9} = 2i \cdot 3i = 6i^2 = -6$$
 is correct,
while
$$\sqrt{-4} \cdot \sqrt{-9} = \sqrt{(-4)(-9)} = \sqrt{36} = 6$$
 is incorrect.



2(a) Multiply or divide, as indicated. Simplify the answer.

 $\sqrt{-7} \cdot \sqrt{-7}$

Solution:

First write all square roots in terms of *i*.

$$\sqrt{-7} \cdot \sqrt{-7} = i\sqrt{7} \cdot i\sqrt{7}$$

$$= i^2 \left(\sqrt{7}\right)^2 = i^2 \cdot 7 = -7 \qquad i^2 = -1$$



2(b) Multiply or divide, as indicated. Simplify the answer.

 $\sqrt{-6} \cdot \sqrt{-10}$

$$\sqrt{-6} \cdot \sqrt{-10} = i\sqrt{6} \cdot i\sqrt{10} = -1\sqrt{60}$$
$$= -1\sqrt{4 \cdot 15} = -1 \cdot 2\sqrt{15} = -2\sqrt{15}$$



2(c) Multiply or divide, as indicated. Simplify the answer.

 $\frac{\sqrt{-20}}{\sqrt{-2}}$

$$\frac{\sqrt{-20}}{\sqrt{-2}} = \frac{i\sqrt{20}}{i\sqrt{2}} = \sqrt{\frac{20}{2}} = \sqrt{10}$$
 Quotient rule for radicals.



2(d) Multiply or divide, as indicated. Simplify the answer.

 $\frac{\sqrt{-48}}{\sqrt{24}}$

$$\frac{\sqrt{-48}}{\sqrt{24}} = \frac{i\sqrt{48}}{\sqrt{24}} = i\sqrt{\frac{48}{24}} = i\sqrt{2}$$
 Quotient rule for radicals.



Example 3 Simplifying a Quotient Involving $\sqrt{-a}$

3(a) Write
$$\frac{-8+\sqrt{-128}}{4}$$
 in standard form $a + bi$.
Solution: $= \frac{-8+\sqrt{-64\cdot 2}}{4}$
 $= \frac{-8+8i\sqrt{2}}{4}$ $\sqrt{-64} = 8i$
Be sure to factor $= \frac{4(-2+2i\sqrt{2})}{4}$ Factor.
 $= -2 + 2i\sqrt{2}$ Lowest terms.



ADDITION and SUBTRACTION of COMPLEX NUMBERS

ADDITION and SUBTRACTION of COMPLEX NUMBERS

For complex numbers a + bi and c + di,

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and
 $(a + bi) - (c + di) = (a - c) + (b - d)i$



Example 4 Addition and Subtraction of Complex Numbers

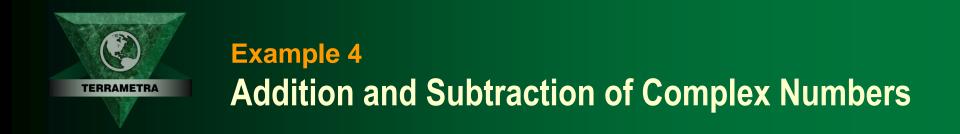
4(a) Find the sum or difference. Write answer in standard form.

$$(3-4i) + (-2+6i)$$

Add Add
real imaginary
parts parts
$$(3-4i) + (-2+6i) = [3+(-2)] + [-4+6]i$$

Commutative, associative, distributive properties.

$$= 1 + 2i$$
 Standard form.



4(b) Find the sum or difference. Write answer in standard form.

(-4+3i) - (6-7i)

Solution:

$$(-4+3i) - (6-7i) = (-4-6) + [3-(-7)]i$$

= -10 + 10i Standard form.



MULTIPLICATION of COMPLEX NUMBERS

MULTIPLICATION of COMPLEX NUMBERS

For complex numbers a + bi and c + di,

(a+bi)(c+di) = (ac-bd) + (ad+bc)i



MULTIPLICATION of COMPLEX NUMBERS

The product of two complex numbers is found by multiplying as though the numbers were binomials and using the fact that $i^2 = -1$, as follows:

$$(a + bi)(c + di) = ac + adi + bic + bidi$$
 FOIL

$$= ac + adi + bci + bdi^2$$

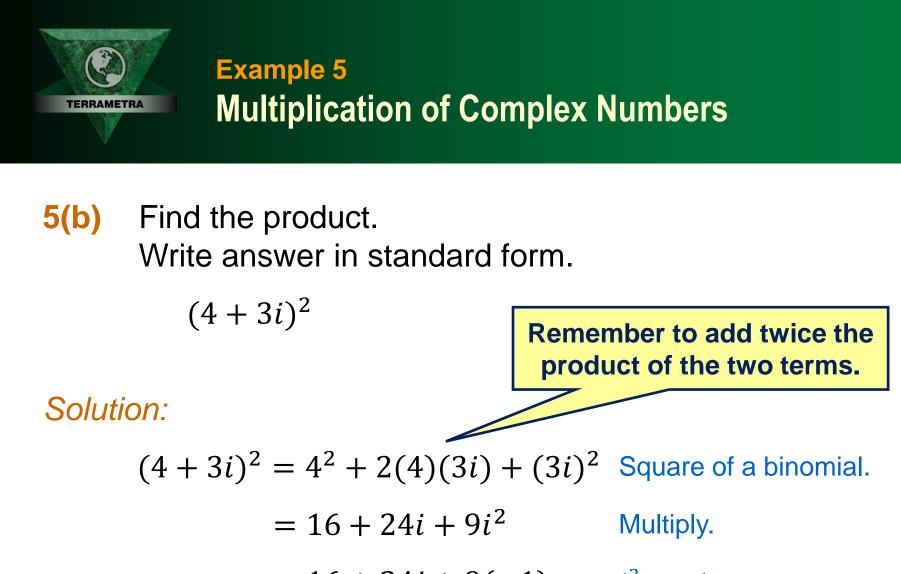
Distributive property. = ac + (ad + bc)i + bd(-1) $i^2 = -1$

$$= (ac - bd) + (ad + bc)i$$



5(a) Find the product. Write answer in standard form. (2-3i)(3+4i)

$$(2-3i)(3+4i) = 2(3) + 2(4i) - 3i(3) - 3i(4i)$$
 FOIL
= $6 + 8i - 9i - 12i^2$ Multiply.
= $6 - i - 12(-1)$ Combine like terms.
 $i^2 = -1$
= $18 - i$ Standard form.



- $= 16 + 24i + 9(-1) \qquad i^2 = -1$
- = 7 + 24i Standard form.



5(c) Find the product. Write answer in standard form. (6 + 5i)(6 - 5i)

$$(6+5i)(6-5i) = 6^{2} - (5i)^{2}$$

$$= 36 - 25(-1)$$

$$= 36 + 25$$

$$= 36 + 25$$

$$= 61 \text{ or } 61 + 0i$$

$$= 51 \text{ Standard form.}$$



Example 5 Multiplication of Complex Numbers

Example 5(c) showed that ...

$$(6+5i)(6-5i) = 61$$

The numbers 6 + 5i and 6 - 5i differ only in the sign of their imaginary parts and are called <u>complex conjugates</u>.

The product of a complex number and its conjugate is always a real number.

This product is the sum of the squares of the real and imaginary parts.



PROPERTY of COMPLEX CONJUGATES

PROPERTY of COMPLEX CONJUGATES

For complex numbers a and b,

$$(a+bi)(a-bi) = a^2 + b^2$$



6(a) Find the quotient. Write answer in standard form.

$$\frac{3+2i}{5-i}$$

Solution:

$$\frac{3+2i}{5-i} = \frac{(3+2i)(5+i)}{(5-i)(5+i)}$$
$$= \frac{15+3i+10i+2i^2}{25-i^2}$$

Multiply by the complex conjugate of the denominator in both the numerator and the denominator.

Multiply.



Example 6 Multiplication of Complex Numbers

Solution (cont'd):

$15 + 3i + 10i + 2i^2$	13 + 13i
$25 - i^2$	26

Combine like terms. $i^2 = -1$

$$=\frac{13}{26}+\frac{13i}{26}$$

 $=\frac{1}{2}+\frac{1}{2}i$

Ī

$$\frac{a+bi}{c} = \frac{a}{c} + \frac{bi}{c}$$

Write in lowest terms and standard form.

Check:

$$\left(\frac{1}{2} + \frac{1}{2}i\right)(5-i) = 3 + 2i$$



6(b) Find the quotient. Write answer in standard form.

Solution:

3

 \overline{i}

$$\frac{3}{i} = \frac{3(-i)}{i(-i)} = \frac{-3i}{-i^2} = \frac{-3i}{1}$$

-i is the conjugate of i
and
 $-i^2 = -(i^2) = -1(-1) = 1$

= -3i or 0 - 3i Standard form.



Powers of *i*

Powers of
$$i$$
 can be simplified using the facts ...
 $i^2 = -1$ and $i^4 = (i^2)^2 = (-1)^2 = 1$